

## The Debate about the Structural Behaviour of Gothic Vaults: From Viollet-le-Duc to Heyman

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**ABSTRACT:** The structural behaviour of the gothic vault will be discussed, focusing in the debate about Viollet-le-Duc's rational theory (ca. 1850): the "active" ribs support the load of the "passive" masonry of the webs. The debate reached its climax in the 1930's with the frontal attack mounted by Abraham: the ribs are merely decorative; it is the shell which carries the vault. Other eminent French scholars (Aubert, Focillon), were not so drastic, since Abraham, Viollet's ideas have been looked with suspicion. The debate is still alive, though in fact was closed by Heyman in the 1960's, when he formulated the principles of the modern Limit Analysis of masonry structures. Within this new theoretical frame it is a false debate, as it states a question which cannot be answered: what is the "actual" or "true" structural state of a building. This fact, discovered in the 1920's by Baker, supposes a Copernican change in the approach to the analysis of structures. The debate on gothic vaults may serve to illuminate this approach and its corollaries.

Gothic vaults, with their geometrical patterns of ribs springing from the shafts of slender columns, have been always a source of wonder. In this paper the only the problem of the structural behaviour of the gothic ribbed vault will be discussed. However, the vault is only a part of the much more complex structure of the gothic churches and cathedrals. The reason to concentrate on the structural behaviour of this particular element is that this question was at the heart of a long debate on the technical interpretation of Gothic; besides, to fix the attention in one element will permit to go to the heart of the problem, which is eventually, the structural analysis of masonry structures.

### THE INTEREST IN GOTHIC CA. 1800

The gothic structure was admired by some architects and engineers (Guarini, Perronet, Soufflot) by its lightness and strength even at the times when the gothic architecture was despised as "barbarian" or simply ignored. However, there were the English antiquarians of the end of the eighteenth Century, who first turned their attention to gothic. At the beginning the attention centred more in the descriptive and picturesque than in the practical or engineering aspects. Britton, Pugin and others began to inventariate and draw the gothic monuments with an increasing level of accuracy and detail. Then, some authors tried to identify the fundamental elements, and there was unanimity in considering the rib as essential to gothic architecture. What is the origin of the rib? This was one of the main the questions. (For a detailed discussion of the first studies of gothic and its essential elements, see Frankl 1960, pp. 489-525.)

The first to write a paper on the geometry of the gothic vaults as derived from the ribs was Ware (1814). Though he considered, wrongly, that the ribs which were not circular were plane projections of circular ribs (i.e. ellipses), his classification of the form of the different vaults and, above all, his wonderful drawings (showing for the first time in perspective the relationship between the plan and the ribs) had great influence and helped to concentrate the attention on the true geometry of the ribs. Shortly after, Saunders (1816) insisted in the crucial role of the ribs in the origin of the Gothic. At the same time, the surveying of the gothic buildings and their elements, the vaults included, were increasingly more and more precise. The plans and elevations of vaults by Pugin (1825) are remarkable for his degree of accuracy: maybe for the first time the radius of the ribs were measured and represented in plans.

## ROBERT WILLIS

Robert Willis was the first to propose a theory on the possible geometric generation of the gothic ribs. Already in his book on the architecture of the Middle Ages in Italy (Willis 1835) he dedicated explicit attention to rib vaults and their mechanical action. Willis was, also, the first to draw the attention to the structural function of the ribs and groins (acting like ribs), as structural lines of transmission of the loads (Willis 1835, p. 70). This observation was made within the context of his distinction between what he called *mechanical construction* (how the weights are really supported) and *decorative construction* (how they seem to be supported) (Willis 1835, p. 15). Ribs, then, may be mechanical or decorative. Mechanical ribs are the "genuine ribs, strengthening and sustaining the vault" (Willis 1835, p. 80). Decorative ribs are needed to give an impression of stability. In 1842 Willis published a long paper of some sixty pages "On the construction of the vaults of the Middle Ages" (Willis 1842), where he concentrated his attention on the constructive and geometrical problems, comparing his theories with the result of some accurate surveyings. In it he insisted in the same ideas: gothic vaults with the ribs constructed independently of the masonry of the webs, he called "rib and panel" vaults, and this is a mechanical construction; on the contrary, when the ribs and panels are carved on the surface of a continuous masonry vault, these are merely decorative (Willis 1842, p. 7). Willis considered, then, of crucial importance to make evident the independence of the ribs from the webs, even in cases of decorative construction, as it occurs with most fan vaults: "the ribs really support the vault, and should appear to do so in the decorative as well as in the mechanical construction" (Willis 1842, p. 25). However, he renounced to enter into the matter of the structural behaviour of the vaults (Willis 1842, p. 67). We found, then, in Willis the nucleus of Viollet-le-Duc's theory of the structural behaviour of gothic ribbed vaults.

## VIOLLET-LE-DUC AND CHOISY

Eugène Viollet-le-Duc, a French architect who worked intensely in restoration, was the most important architectural theorist of his time and constructed a whole theory of the gothic. Within this "système archéologique" ordered within a rational framework, the technical and constructive aspects were a fundamental part, and within them there was an explicit interpretation of how a gothic ribbed vaults behaves. In his first articles published in the *Annales archéologiques* between 1844 and 1847 we find already expressed his theory of the behaviour of the vaults (Viollet-le-Duc 1844-1847, pp. 143-150). The same ideas were repeated and expanded in his *Dictionnaire* published in 1854-1868 (I am using the reprint of 1875) which exerted an enormous influence in successive generations of medieval archaeologists (Frankl 1960, pp. 565-578).

For Viollet-le-Duc the origin of the rib lies on the necessity of a centering for transverse arches and the groins of the groined vaults. The wooden centrings were replaced by stone-centrings, the *voussoir* ribs. The ribs support the masonry of the webs during the construction, and, after its completion, the whole weight of the vault, facilitating also the difficult bonding of the stones at the groin. Finally, as the loads are entirely transmitted by the ribs they will be concentrated at certain points and, what more logical than to put an inclined flying buttress to transmit the inclined thrust of the vault to the external buttresses?

There is another aspect which Viollet-le-Duc considered crucial: the *élasticité*. The term should be understood as the capacity to adapt to the settlements and movements of the supports of the vault and not in his usual acceptation. This property was a consequence of the use of the ribs, composed of numerous *voussoirs*, and their not being interlocked with the masonry of the webs. The idea came to his mind studying the deformations of the barrel Romanesque vaults with gross transverse arches (Viollet-le-Duc 1875, p. 14), which he was the first to describe in detail (Viollet-le-Duc 1875, IV, p. 21). Finally, it should be noted another aspect of his structural interpretation: the active nature of the internal forces in a gothic structure and the crucial role presented by the equilibrium. Viollet is, in fact, suggesting that the structure of the gothic building, as the ribbed vault, has the same property of *élasticité*, the capacity to adapt to different situations of loads and changes in the boundary conditions (Viollet-le-Duc 1875, IV, p. 127).

Viollet-le-Duc's comments on the function of the rib were dispersed through his *Dictionnaire*. In fact it was Auguste Choisy who expanded and systematized Viollet's ideas. In his book *Histoire de l'architecture* he dedicated a whole section to the gothic vaults and within this he tackled the matter with outmost clarity (Choisy 1899, pp. 267-270). Choisy, who have studied in detail the geometry and construction of Roman and Byzantine vaults, was in the best position to do this and exposed a rational explanation of gothic vault construction, which agreed mostly with Viollet. It is worthwhile to read the paragraph concerning the structural action of the vault, "Aperçu des efforts développés para la voûte gothique", from which is the following quotation: "Les nervures, exécutées en plus grand appareil que les panneaux, tassent moins et forment dans le corps de la voûte comme des raidisseurs qui prennent pour eux la majeure partie de la charge et la convertissent en poussées; et ces poussées se propagent suivant les plans verticaux des nervures." (Choisy 1899, p. 269). After Choisy it was not possible to ignore the question, Fig. 1 (a): "On saisit au seul aspect de la fig. 2 la nature des efforts qui se développent". (Choisy 1899, p. 268). However, it should be noted that Choisy is not saying that the whole weight is carried by the ribs, but that they carry the major part of the weight, due to their ashlar construction in contrast with the rubble masonry of the webs.

**THE MECHANICAL STUDY OF THE GOTHIC VAULTS IN THE NINETEENTH CENTURY**

In all the nineteenth Century discussions about the structural behaviour of the gothic vault and the function of the ribs there is no mention about the mechanical, scientific, study of these masonry structures. The first structural analysis of gothic vaults was made in the second half of the nineteenth century. The theory of the arch was by then well developed and, for example, arch bridges were calculated by engineers routinely, and the concept of "line of thrust" improved enormously the understanding of arch statics; eventually the graphical methods of drawing thrust lines were in common use, say after 1870, to check the stability of arches and barrel vaults (Huerta 2004, pp. 523-532).

The study of spatial vaults or buildings was much more complicated. In the case of a barrel vault may be imagined as "sliced" or composed by several independent arches of uniform depth, and the same tools for arch analysis were applied to barrel vaults. This idea of the sliced barrel vault was used by Scheffler (1857, pp. 176-184) to study the statical behaviour of groined and cross vaults: the two barrels were cut in elementary arches, of diminishing size as they approach the centre, which were supported by the groin arch or the cross rib (Huerta 2008, p. 309). Scheffler's calculation involved complicated algebraical formulations and was never used in practice, but it showed the path to the analysis of cross vaults by means of graphical statics. The first practical analysis was made by Wittmann (1879), Fig. 1(b) and Planat (1887), Fig. 1(c), but it was Karl Mohrmann in his new edition of Ungewitter's third edition (Ungewitter, Mohrmann 1890) who made an extensive use of the technique. In fact he did not confine himself to the vaults but made an analysis of the whole building, including flying buttresses, columns, external buttresses, etc. Mohrmann's work is still the more complete study of the statics of a gothic church or cathedral.

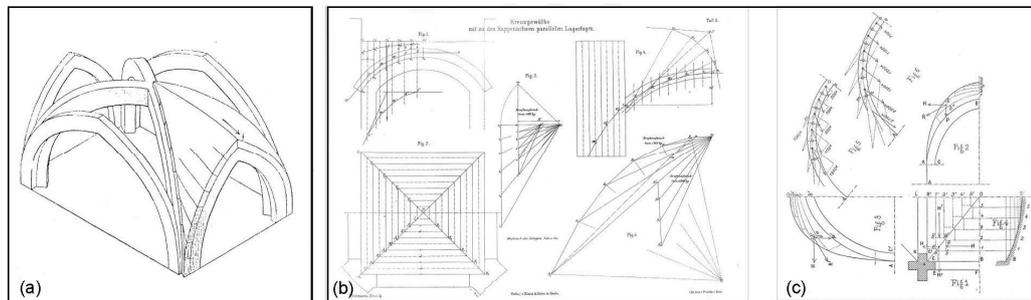


Figure 1: (a) Gothic vault; (Choisy 1899, p. 268); (b), (c) Graphical analysis; (Wittmann 1879), (Planat 1887)

Mohrmann introduced many original approaches to simplify the analysis. Concerning the "slicing" technique, if the cross vault is formed by the intersection of two barrels, the cutting planes are evident. However, Mohrmann considered more complicated forms and combinations of vaults, and, also, considered domical webs. Then, he felt compelled to look for some law to decide the family of cutting planes which will divide the web in elementary arches. He considered that the forces will follow a path similar to that followed by a ball rolling down on the extrados of the webs, Fig. 2. The idea gave him a simple way to imagine the pattern of cutting planes. This idea was used later by Sabouret (1924) and Abraham (1934) who made expressive drawings to explain it. As neither Mohrmann nor Sabouret made explicative drawings, the credit for this idea has been usually given to Abraham. Once decided the elementary arches, drawing the line of thrust within the diagonal arches or groins was a simple matter (Fig. 2 (a), right).

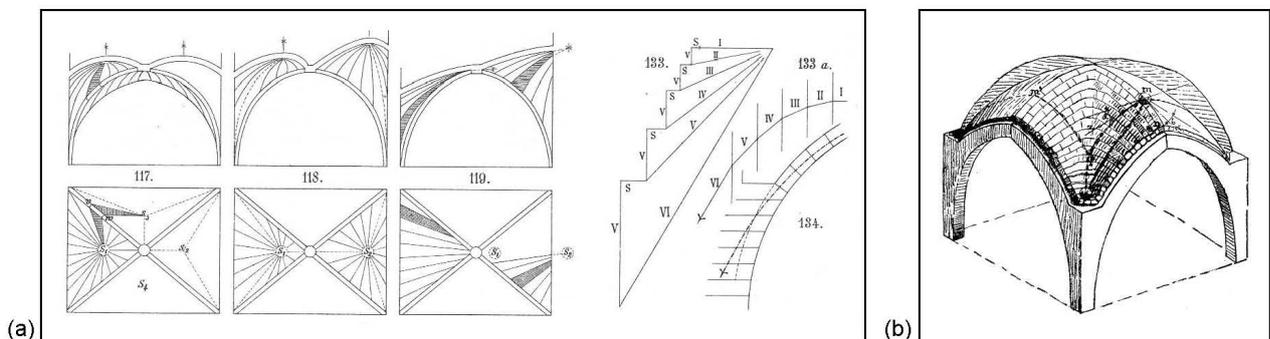


Figure 2: (a) Patterns of slicing (Ungewitter 1890, T.15 ); (b) Path of forces: "ball principle" (Abraham 1934, p. 33)

Therefore, circa 1900, architects could study the statical behaviour of vaults or vaulted buildings with the help of the simple methods of graphical statics. This was made, for example, by Benouville (1890) for the cathedral of Beauvais and y Rubió and Bellver (1912) for the cathedral of Palma de Mallorca.

## THE CRITICS OF VIOLLET-LE-DUC

The criticism to Viollet-le-Duc's archaeological system came very soon. For example, just a few one year after his death, Anthyme de Saint Paul wrote an extense critic of more than three hundred pages. Other French archaeologists (Brutails, Vaillant) followed this general critic to Viollet's *système archéologique*. However, they did not criticize his theory of vault behaviour. It appears that the first critic to the structural role of the ribs came from an American archaeologist, Arthur Kingsley Porter: he saw in the ribs only the solution of the problem of economical centering, and introduces some reinforcement to the groin (Porter 1911, p. 16). This view was accepted by some other authors. In 1920, Roger Gilman tried to extract conclusions as to the state of the theory of gothic through the observations of the damages caused by shellfire during the First World War. Again, he is very critical of some rationalist interpretations, but on the function of the ribs he mainly agrees with Viollet-le-Duc (Gilman 1920, 59).

The first serious attack came from a French engineer, Victor Sabouret, with a solid formation in applied mechanics and an extensive experience in bridge design. In 1924 Sabouret published a paper whose title was a provocation to the orthodoxy of gothic: "Les voûtes d'arêtes nervurées. Rôle simplement décoratif des nervures" (The ribbed groined vaults. Role merely decorative of the ribs). The article by Sabouret is systematic and he exposes his ideas with great clarity. First he comments the geometry: he imagines the vault generated by a cylindrical barrel vault intersected by transversal barrel vaults. The proportions of the web are square at the crossing, but more often the longitudinal barrel has greater span, leading to rectangular compartments. He then studies the barrel vault. The material masonry should work in compression (tensile strength is negligible) and the sliding failure is very rare. The impossibility of sliding permits the formation of hinges which he calls "joints de rupture". Up to this point the vault pertains in reality to the wall, and the space between the extrados and the wall is filled with solid masonry. This reduces the surface of the vault.

He considers two modes of equilibrium. In the first, for rectangular bays, he supposes that the thrust may be diffused radially and makes for this an analogy with the skew vaults, *voûtes biaises*. To analyse a skew vault it is imagined to be sliced by planes parallel to the front arch; if we consider a trapezoidal form, we may assume radiating slicing planes. The limit is the angle of friction which must not be surpassed (he considers  $30^\circ$ ). In this hypothesis, there is no need for ribs.

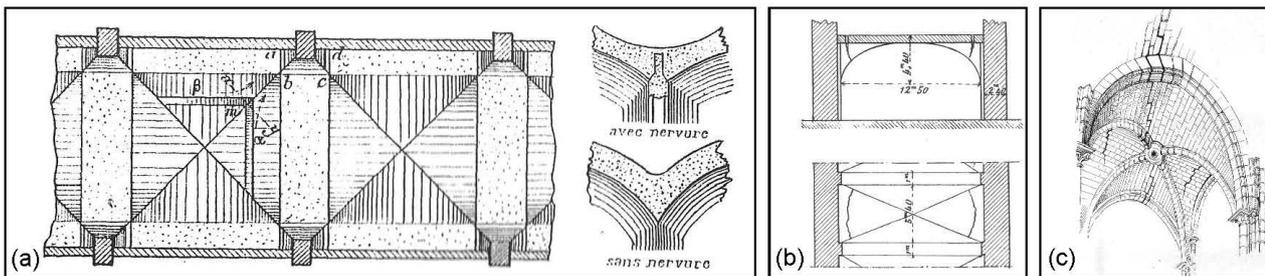


Figure 3: (a) Structural analysis and Cracking of a cross vault; (Sabouret 1928, pp. 208, 209));  
(c) Typical cracking; (Abraham 1934, p. 32)

This mechanism can not be considered for greater angles, and, in this case, he returns to the slicing of the barrels in elementary arches, which are supported by the "arch groin". This arch groin can be formed within the thickness of the vault and, again, there is no need for the ribs. In any case, says Sabouret, the dimension of the ribs is usually too small to be of any importance: "une nervure d'arêtier, collée ou pénétrante, ne représente jamais qu'une faible fraction de la section de maçonnerie où se transmet la poussée résultante vers le pilier" (Sabouret 1928, 199-200), Fig. 3 (a). Besides, Sabouret discards the role of the ribs as permanent centering (Porter's main function of the rib; see above), due to the small spans of the church vaults (less than 25 m!; this was a small span for a bridge. We see here the civil engineer) which render the device unnecessary. Eventually, to finish his argument he says that in many cases the ribs are separated from the groin, due to the movements of the vault. He examines three cases, and though, the argument in relation to the ribs is not very clear, it is, in any case, the first description of the typical cracking of groined and ribbed vaults, Fig. 3 (b). After all this, the conclusion is evident: the role of the ribs is merely decorative.

Sabouret article inspired the structural arguments in Pol Abraham's dissertation on "Viollet-le-Duc et le rationalisme médiéval", began in 1923 and eventually published in 1934. But Abraham, also, have studied with detail Paul Planat's handbooks with numerous examples of masonry vault and building analysis (Planat 1887, 1906). With this "arsenal" Abraham mounted a formidable attack to the rational approach to gothic architecture, and he used for this every argument at his disposal. For him the ribs are decorative, the flying buttresses are useless, the pinnacles have no structural action, and so on. His reasoning is not always correct, but the detail of his analysis (almost any significant phrase in the *Dictionnaire* is subject to the strictest scrutiny), the numerous explanatory drawings, and his deep scholarship (he seems to have read all the everything about the rational approach to gothic published in French; there are no references to English or German literature) convinced the successive generations of the general falsehood of the theories of Viollet-le-Duc and Choisy. His drawings of Mohrmann's rolling ball to define the direction of the thrust (see Fig. 3 (b) above) of the functioning of the groined, ribless, vault, and of the cracked state of a gothic vault, have been reproduced once and again in

books and articles on the analysis of gothic structure (Abraham 1934, pp. 28, 32). And yet, there is nothing basically new in his arguments. Only the exacerbated tone results remarkable. Notwithstanding this, it is a book that should be studied carefully by any student of gothic structural theory.

Other eminent French archaeologists had a more equilibrated view. The long essay of Marcel Aubert (1934) on the first cross vaults, besides being a mine of information, contains many interesting discussion on topics which require a calm and detailed analysis. Focillon (1939) deserves also a rereading. Others defended again, with other arguments, the structural function of the rib (Masson 1935). However, since Abraham's book most scholars (for example Rave (1939, 1955), in Germany, and Torres Balbás (1939, 1945) and considered that the function of the rib was under suspicion. Frankl (1960, 810) seems to have shared, in the end, the same doubts.

#### THE SITUATION AFTER THE SECOND WORLD WAR

The Second World War produced in Europe an enormous destruction. Historical monuments were not saved, and many gothic cathedrals and churches had to be consolidated or rebuilt in part. In many cases the vaults fell down leaving the walls, and the scenarios reproduced by Gilman (1920) repeated again. Usually, the churches were rebuilt in the same form and with the same materials as before the bombardments, without any analysis or calculation. The argument was clear: if the buildings so constructed have stood for centuries, rebuilt in the same way, they will have the same enormous degree of safety. Then, in contrast with the situation today, there were still many masons familiar with the procedures of traditional masonry building.

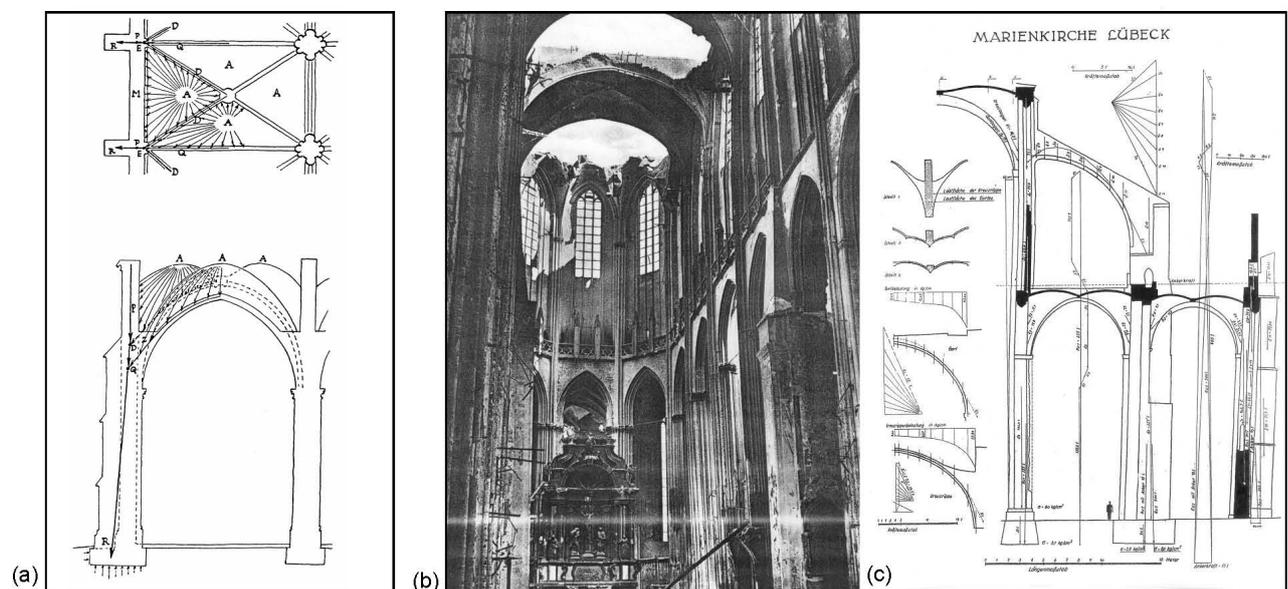


Figure 4: (a) Equilibrium solution represented by thrust lines; (Rave 1939, p. 195); Marienkirche, Lübeck; (b) State after the bombardments; (Pieper 1983); (c) Statical analysis of the Marienkirche, Lübeck; (Pieper 1950, p. 601)

Sometimes the situation of the ruin was so critical that a structural analysis was needed. Of course, by then the elastic analysis was unanimously accepted among theoreticians of structures as the only correct approach. This elastic analysis was possible with arch bridges, but completely impossible for the complex structures of the historical masonry buildings. There was, also, not much time to think or develop new analytical techniques, as the situation was really critical in many instances. In these cases, the same techniques of graphical analysis of vaults discussed above were used, and the handbooks of Planat and Mohrmann were again consulted, and the usual explanation of the behaviour involved the slicing technique and the drawing of thrust lines (Rave 1939). The case of Klaus Pieper deserves to be mentioned. He reconstructed and consolidated many German churches and his method was to use graphical equilibrium analysis, considering the real geometry with its leanings and distortions, and the real state of the masonry, with its cracks. He began his work consolidating the Marienkirche in Lübeck (Pieper 1950), Fig. 5 (a), and his life long experience was compiled in his book *Sicherung of historischer bauten* (Pieper 1983), Fig. 5 (b). Other case which deserves to be mentioned, due to the extensive and detailed statical analysis made, is that of the cathedral of Xanten (Grassnick 1963).

#### THE ELASTIC SOLUTION: FROM PHOTOELASTICITY TO FEM COMPUTER PACKAGES

Though in the practical works of restoration the exigency to give fast and concrete answers "forced" the use of equilibrium analysis using often the tools of graphical statics, the approach still was considered as approximate or downright false by many professors and engineers. The correct approach was to obtain the elastic solution (as it was made in the case of masonry arches at the end of the nineteenth century), solving the system of equations of equilibrium, material and compatibility. In the 1960's an indirect approach was followed: the use of photoelastical methods. The first to apply these methods to gothic cathedrals was Robert Mark (1968).

Then, in the 1970's he combined the use of spatial models with the employment of Finite Element programs trying to extract conclusions of the behaviour of gothic vaults and structures and he published many articles on the subject. Using both techniques Mark has studied the gothic ribbed vaults. The results were eventually compiled in a book (Mark 1982). Other authors, for example Kübler (1974), followed the same elastic FEM methods. Today, the use of Finite Element computer programs it is considered by many historians and engineers as the best tool to investigate the behaviour of historical masonry constructions (see, for example, Coste 1997).

It is hard to think of any historical masonry construction as made of a continuous, isotropic elastic material. It is even harder to consider the possibility of knowing the compatibility equations, both internal (the manner in which the different elements are connected) and external (the boundary conditions), which control the geometry of deformation. It is true that, at least, the equilibrium equations are universal and that the program is giving a possible solution, between the infinitely many possible in such highly hyperstatic structures. But to suppose that this solution is the "actual" solution, that it represents the "real state" of internal forces, is to show a degree of naïveté that no engineer or architect, indeed any serious student of historical constructions, can afford. As professor Heyman has pointed out many times, it is a fact that the system of the equations of equilibrium, material (constitutive) and compatibility, is extremely sensible to small changes in the compatibility conditions, particularly in the boundary conditions (see for example, Heyman 2008). This may be readily checked using the same FEM packages: a settlement of a few centimetres in a column, a leaning of 1° of a buttress, will distort completely the "actual" state obtained some minutes before. As it is impossible to know the internal and external compatibility conditions, which, besides, change with time, the classical elastic approach is nonsensical.

The use of non-linear FEM analysis, with a simulation of a material which has no tensile strength, is, of course, much better. But the fact remains that the obtained solution may not represent the "actual" state of the structure. The system of equations remains to be very sensible to the original boundary conditions and is also sensible to the history of loading. However, an experienced engineer may extract interesting results from the use of non-linear FEM packages. This is the case, for example, of Barthel (1991) who made a comprehensive study of the behaviour the possible crack patterns in cross vaults, combining the use of the computer, with a deep scholarship on the matter and a wide practical experience in restoration works.

The question, then, remain unanswered: What is the actual structural behaviour of a gothic vault, has the rib any structural function?

## HEYMAN'S THEORY OF LIMIT ANALYSIS OF MASONRY STRUCTURES

The debate on gothic vaults leads us to a crucial problem in the theory of structures. What is the actual state of a structure? This problem is in the origin of a new theory of structures. At the beginning of the twentieth century there was a general agreement in that the only correct analysis was the classical elastic analysis (even for masonry arches and buildings). Nobody has questioned the mathematical apparatus and the rigour of the elastic approach. Then, in the 1920's a set of experiments on framed real structures were made with the intention to improve the design codes of practice. The result was that the calculated stress resultants (bending moments) have almost nothing in common with the actual bending moment distributions observed measuring the curvatures of the elements. This discovery was the origin of the Plastic (or Limit) Analysis of Steel Frames, developed in the 1940's by Baker et al. However, if the actual state was impossible to know, it was possible to calculate with great precision the collapse load of the structure (which was insensitive to small changes of boundary conditions) and therefore the *safety* of the structure. Three Fundamental Theorems were demonstrated which allowed to ascertain the safety of any "ductile" structure with absolute rigour (Heyman 1998, pp. 127-52). The Safe Theorem states that if it is possible to find a set of internal forces in equilibrium with the loads, which satisfies the yield condition, then the structure is safe. Heyman was the first to see the all important corollary contained in the Theorem: This set of internal forces need not to be the "actual" one, it is enough that it is possible. This leads to the "equilibrium approach": we may consider any solution which respect the yield condition of the material and can consider the boundary conditions as unknown (Heyman 1998).

In the field of masonry structures, Heyman (1966) realized that the same theoretical frame could be translated to masonry as long as the material satisfied three conditions: infinite compressive strength, no tensile strength and impossibility of sliding failure. (Precisely the usual conditions accepted in the nineteenth century for masonry arch analysis and cited by Sabouret (1928, p. 206).) The Fundamental Theorems applied also to masonry. For convenience, let us consider a masonry arch: In a masonry arch a line of thrust represents a possible set of internal forces in equilibrium; if the line is contained within the masonry, the yield condition is satisfied, and the arch is safe. If we can draw a line of thrust within the arch, it will not collapse, Fig. 5 (a). Of course, any little movement of the abutments will produce a certain cracking and a change in the position of the line of thrust, but due to the Safe Theorem it will never go out of the masonry. The demonstration requires higher mathematics, but cardboard models of arches may be also used for a simple experimental verification, Fig. 5 (b). (Huerta 2005). The same apply to any masonry structure. Heyman (1967, 1977) have studied the rib problem in detail, and there are sound reasons to consider, in general, a sudden increase of the stresses at the rib. However, any conclusion about the real state of a structure should be taken with care, as Heyman has pointed out once and again. Let us consider the vault and buttress system of Fig. 4 (a) and suppose a span of 10 m; a settlement of, say, 100 mm of the right column, will produce the cracking of the vault, which will adapt to the movement. However, the equilibrium solution is still valid as the settlement is only 1/100 of the span, more or less the thickness of lines on the drawing (Heyman 1995, p. 23). But the state of the ribbed vault may change completely:

ribs may separate from the vault shell (or a previous crack may close), the webs would crack limiting the paths of forces. On the other hand, the bad state of the roof for half a century may have allowed the entry of water producing the deterioration (even the desegregation) of the mortar joints. Sometimes, even, a voussoir of one of the ribs may fall, or a stone of the webs. . . . The ribs may carry a part of the load, or no load at all, at different times on the history of the monument. And the same occurs with the shell of the vault webs between the ribs. One may find the examples that best suit the preferred "theory" (Fig. 5 (c), (d)).

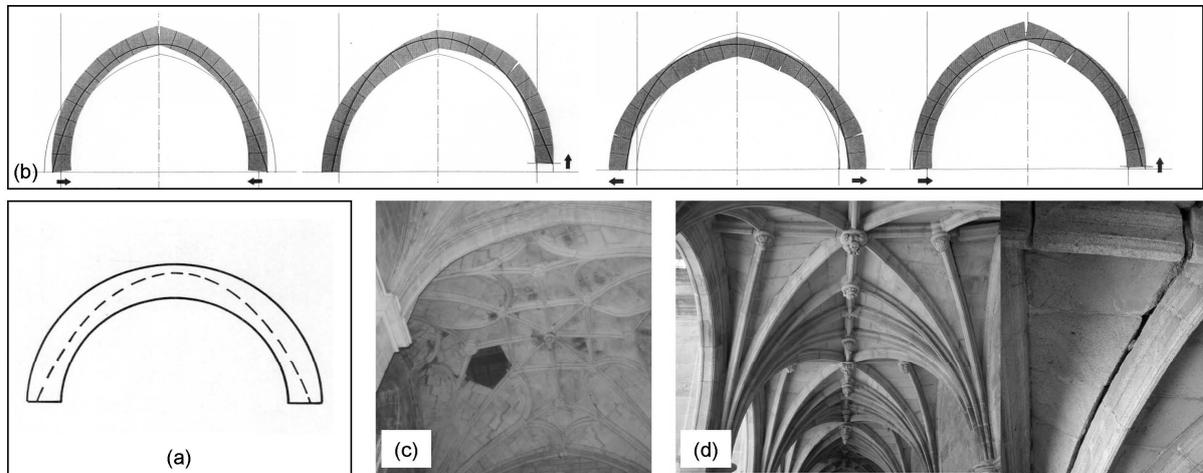


Figure 5: (a) Arch in safe equilibrium; (Heyman 1995, p. 21); (b) Crackings in arches due to small movements of the abutments; (Huerta 2005, p. 755). (c), (d) Actual states of different Spanish gothic ribbed vaults

## CONCLUSION: THE END OF THE DEBATE

Many debates have their origin in a false premise, in stating the wrong question. In this case the false premise was (and is) that it exists an unique, *actual*, state of internal forces for a certain structure under certain loads, and the wrong question was to ask what is this "actual" structural behaviour of the gothic vault or building, if the rib carries or not, if the flying buttresses and pinnacles are decorative, if the column shafts have any structural meaning, etc.

Indeed, the great discovery of the theory of structures in the twentieth Century is precisely that this premise is false and that this question has no sense. It is not possible to know if it is the rib or the shell of the intermediate webs which carries the vault loads. In most cases both elements transmit a part of the total load, but the proportion may change with time, as has been discussed above. In some cases, a detailed study of the pattern of cracks could give some indications of the state at this time of the history of the monument. But the passing of time, the successive interventions and events in the life of the building, will no doubt modify this state. However, the safety is no affected by these sudden and sometimes enormous changes in the distribution of internal forces. The stability in masonry buildings is a matter of the overall geometry of the building and remains unaffected by little movements or changes.

It is the great contribution of professor Heyman to the study of gothic, to make us aware of the consequences of the modern theory of Limit Analysis of structures for the structural interpretation of gothic architecture. That there is no "close" or "unique" solution does not mean that buildings are impossible to understand or analyse. The historian, architect or engineer would take one approach or another depending on the problem. The search is for reasonable states of equilibrium in compression which could help to throw light on the problem or question at hand. The freedom to investigate different ways enriches the field in an extraordinary way and liberates the mind of the analyst in his search for answers to meaningful questions.

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