

Modelling Tools for the Mechanical Behaviour of Historic Masonry Structures

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ABSTRACT: The study presented herein has been performed in ARIAM-LAREA and focuses on the modelling and the analysis of the mechanical behaviour of historic masonry structures. Initially, we present a tool of interactive evaluation of the thrust of a masonry structure. To this end we use a program of dynamic geometry and apply some fundamental concepts of graphic statics. A tool for the evaluation of the stability of typical structural systems encountered in medieval structures is, also, presented for dead-loading and for pseudo-static seismic loading and it is applied to the analysis of the medieval church *Santa Maria del Bourgo* of Rhodes, Greece. This approach, based on limit analysis, is particularly suitable for two-dimensional or pseudo three-dimensional configurations of structures and loadings.

INTRODUCTION

Historic masonry structures are significant parts of the world cultural heritage and exhibit a large diversity of geometries and building materials. Because of their importance, the analysis of their stability constitutes a ubiquitous question. The study presented herein has been performed in ARIAM-LAREA (research laboratory of the ENSAPLV) and focuses on the development of tools allowing for the modeling and the analysis of the mechanical behavior of historic masonry structures. The tools aim at responding to the needs of engineers and architects as regards the preservation of historic and monumental buildings, and they are adapted to the intrinsic mechanical behavior of the building materials of old structures. The presented models are oriented towards the following goals: a) provide a coherent framework for the description of the mechanical behavior of historic masonry structures, b) serve as an investigation tool for the researcher of construction and building technology allowing for realistic simulations of the structure and permitting the validation of the evolution of pathology of the structure and c) guide the selection of the preservation/restoration technique that is best suited for the structure. This study is the evolution of previous works conducted in ARIAM, concerning a simulation tool for the behavior of gothic domes intended to be used by architects responsible for the historic monuments [Chassagnoux et al. 1998]. The research project has resulted in the development of a prototype allowing the architect to create a mesh of the studied structure for a finite element calculation [Chassagnoux et al. 2000]. The tools presented in this paper are based on the theory of limit analysis and simulate the behavior of the structural elements or two-dimensional sections of structures obtained from 3D model restitution software developed in ARIAM.

LIMIT ANALYSIS OF MASONRIES

Stability of masonry structures: problem statement

For the majority of masonry structures, the problem of stability is mostly related to the geometry of the structure rather than the resistance of the building materials (Heyman 1995). The traditional rules of ancient builders for the design of masonry domes and flying buttresses were geometrical, in the sense that they defined the proportions between the structural elements. Until the eighteenth century, even though builders possessed neither scientific construction methods nor knowledge of the materials resistance and the underlying mechanics, they however built the objects of virtue of the gothic architecture (Huerta 2001).

Nowadays, this problem can be formulated within the context of Yield design theory (Salençon 2002). This theory allows determining if a structure can support a given combination of external loads and results in an ap-

proximation of the ultimate loads of a structure from below (static approach) and from above (kinematic approach). Three elements are necessary in order to apply one of the two approaches of Yield design theory: a) the geometry of the structure, b) the loading process and c) the resistance criteria of the material and the interfaces of the structure.

Regarding masonry structures, their geometry-related stability is an immediate consequence of the resistance criterion of structural masonry. As a matter of fact, masonry, viewed as a homogenized material, exhibits a very small tensile strength (practically zero) and a very high compressive strength, usually much larger (practically infinite) compared to the stress states commonly imposed within a masonry structure.

This criterion thus implies that every point in the structure must be subjected to a compressive stress state. Furthermore, if the structure is composed of many blocs that are in compression and if the compression resistance is considered infinite, the only condition to satisfy is the equilibrium of every bloc as a rigid body and this constitutes a problem of purely geometrical nature. In the sequel we propose an analysis method that is based on this property of masonry structures.

Brief outline of existing works

During the antiquity and until the eighteenth century, builders were constructing masonry structures only by experience. Once the relevant mathematical tools were sufficiently developed, studies were conducted aiming at a rational design of masonry structures. Coulomb (1773) noted clearly that the stability of the structure can be assured if the line of thrust lies in the interior of the structure. He proceeded in a calculation of the minimum and the maximum horizontal thrust that can ensure the stability of an arch.

The principal difficulty for treating the problem of the stability of masonry structures is that they are statically indeterminate. In order to overcome this difficulty, Poncelet (1852) proposed to treat masonry as an elastic material. During the same period, many graphical methods were developed for the design of masonry structures (Culmann 1866, Pirard 1967). However, it is the advent of plasticity theory and limit analysis that led to a rigorous formulation of the stability problem of masonry structures.

Heyman (1966) introduced the no tension material and applied the theorems of limit analysis to study the stability of domes and flying buttresses. O'Dwyer (1999) proposed a method complying with the static approach of limit analysis in which the principal stresses of the structure are modeled as a discretized force network. From a theoretical point of view, Huerta (2001) showed that the experience of ancient builders can be interpreted through the modern theories of plasticity and limit analysis. One can finally note the treatment of masonry structures with advanced numerical approaches, such as the discrete element method (cf. the computer code LMGC90 developed by F. Dubois and M. Jean). This last method models a masonry structure as an assemblage of rigid bodies by taking account of the contact conditions (unilateral contact, friction etc) between them.

DYNAMIC GEOMETRY AND LIMIT ANALYSIS

A tool for the evaluation of the stability of masonry was developed by means of dynamic geometry interactive tools. It has been the subject of collaboration between ARIAM and the Department of Architecture of the Massachusetts Institute of Technology (Block et al. 2006a). The introduction of principles of graphic statics within a dynamic geometry environment, allows evaluating the stability of structures in a parametric way by changing specific geometrical features of the structure. For example, the transition from the semi-circular to the pointed arch and the consequences regarding the maximum and minimum horizontal thrust supported by the arch have been studied using the presented tool by Ciblac (2006).

PROPOSITION OF A TOOL FOR THE ANALYSIS OF MASONRY STRUCTURES

Analysis of systems

In order to apply limit analysis to masonry structures the following assumptions will be considered: masonry has no tensile strength; it has infinitive tensile strength; and no sliding can occur within the masonry. The tool for the analysis of masonry structures presented herein has been developed following the method proposed by O'Dwyer (1999). The structure is discretized in a number of blocks that are then reduced to nodes of a network. The gravity centers of each block determine the horizontal coordinate of each node, which remains fixed all through the analysis. The constraints derived from the static equilibrium of each node under a given loading (equilibrium of forces in the horizontal and vertical direction and equilibrium of moments) are formulated with respect to the vertical coordinates of the network nodes. The analysis consists in varying the vertical coordinates of each node until equilibrium is satisfied. Once the position of the nodes has been determined, the line of thrust of the structure can be constructed. As the nodes of the network represent the blocs of the structure in equilibrium, this line visualizes the path of the resultants of compressive forces within the masonry. Loadings considered in O'Dwyer (1999) include the unit weight of the structure (calculated by its geometry), the horizontal reaction force applied at the extremities of the structure, and also an exterior vertical loading applied to the gravity centers of the structure.

This method has been implemented for the analysis of two-dimensional masonry structures. The applications presented in this article refer to the analysis of structural systems that consist of a central arch and two side

arches supported by two piles and two buttresses respectively. By changing the geometry parameters of each element different configurations can be obtained and analyzed and the influence of the structural elements to the behavior of the whole structure can be evaluated (Ciblac et al 2008).



Figure 1: General view of the ruins of the church *Santa Maria del Borgo* (left) and view of the transverse section that connected the sanctuary with the nave (right)

As a case study, the medieval church Our Lady of Burgh (*Santa Maria del Borgo*, Fig. 1) in Rhodes, Greece will be examined. This is a typical example of Hospitaller Rhodian architecture with influences from the rich local Byzantine tradition. It is a three-aisled basilica with two arcades, each consisting of four columns. The nave that had been probably covered by gothic cross vaults collapsed, possibly in 1522. The tripartite sanctuary is still standing today. The transverse section that was connecting the sanctuary to the nave will be examined in the following.

The system comprises three pointed arches and it is symmetric with reference to the vertical axis Oz (Fig. 2). It will be assumed that the strength of the interfaces at the bases of the two piles and the two buttresses with respect to both normal and tangential tractions is infinite. The center point O_{ext} of the external side of the arch is lying above the center point O_{int} of the internal side of each arch.

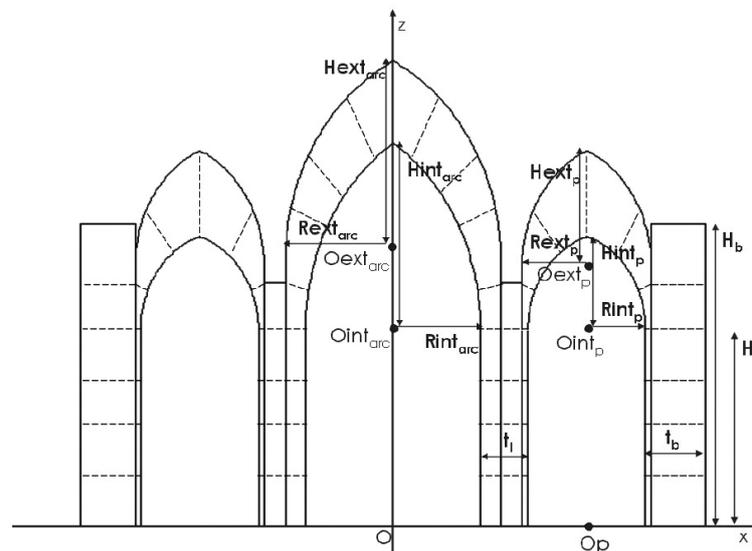


Figure 2: The examined system

The geometry of the system is defined by:

- a) the radius R_{int_arc} of the internal contour of the central arch,
- b) the height H_{int_arc} of the internal contour of the central arch,
- c) the radius R_{ext_arc} of the external contour of the central arch,
- d) the height H_{ext_arc} of the external contour of the central arch,
- e) the radius R_{int_p} of the internal contour of the side arch,
- f) the height H_{int_p} of the internal contour of the side arch,
- g) the radius R_{ext_p} of the external contour of the side arch,

- h) the height H_{ext_p} of the external contour of the side arch,
- i) the height H_l of the piles,
- j) the height H_b of the buttresses,
- k) the thickness t_l of the piles,
- l) the thickness t_b of the buttresses and
- m) the distance between the center point O_{ext} and the center point O_{int} of each arch.

In the sequel geometrical proportions of the real structure are given.

$$\frac{Hint_{arc}}{Rint_{arc}} = 2.16, \frac{Hext_{arc}}{Rext_{arc}} = 1.76, \frac{Rext_{arc}}{Rint_{arc}} = 1.23, \quad (1)$$

$$\frac{Hint_p}{Rint_p} = 1.41, \frac{Hext_p}{Rext_p} = 1.67, \frac{Rext_p}{Rint_p} = 1.10, \frac{t_l}{t_b} = 0.75, \frac{H_b}{H_l} = 1.53, \quad (2)$$

$$\frac{OOint_{arc}}{H_l} = 1.00, \frac{OOint_{arc}}{OOext_{arc}} = 0.74, \frac{OOint_{arc}}{OOint_p} = 1.00, \frac{O_p O_{int_p}}{O_p O_{ext_p}} = 0.76 \quad (3)$$

In all subsequent analyses (both under self-weight and seismic loading), the structure is subjected to in-plane loading. Similar systems with semi-circular arches have been examined by Block et al. (2006a). The method presented by O'Dwyer (1999) is herein adapted to a consideration of the stability of the joints between the arches and the supporting piles/buttresses and also of the entire system under seismic loading. It is noted that examining the stability of arch joints is essential in understanding the coupling between the elements in the examined typical gothic architecture section. Similar considerations regarding the stability of joints can be found among others in the early works of Breymann (1866). The originality of the presented method is that it treats the joint as a separate element, thus overcoming the ambiguity concerning the joint stability when structural elements alone are examined (cf. Heyman 1966 with reference to the works of Breymann 1866).

The goal of the presented analysis is to construct a static solution for the stability of this system expressed through a thrust line that is traced for the structural elements. The points where the thrust line touches the contour of the structure are susceptible to the formation of hinges. Consequently, the opposite points of the outline of the structure are susceptible to crack formation. The thrust line of the structure is thus a simplified means of visualizing the possible areas of damage within the structure.

The static solution for the structure is constructed by assuming a value for the three unknown horizontal forces H_{arc} , H_{pR} and H_{pL} that are applied at the extremities of the central, the right and the left side arch respectively.

Loading under self-weight

Initially, the structure will be considered only under the loading of its own weight. Since the geometry of the structure and the loading are both symmetric with respect to the axis Oz , the horizontal forces H_{pR} and H_{pL} of the side arches will be considered equal to each other: $H_{pR} = H_{pL} = H_p$. The knowledge of these forces is sufficient in order to construct the line of thrust for the entire system.

Assuming a particular value for these forces, two cases are possible:

- a. The line of thrust lies in some region outside the structure contour. This is an unstable state for the structure.
- b. If the line of thrust is everywhere inside the structure: the structure is stable for the combination of the considered forces.

The problem of structural stability is thus reduced to the determination of the values of the forces H_{arc} and H_p for which the system is stable.

In order to achieve this, the following procedure is performed:

1. The central arch is examined separately and the minimal $\min H_{arc}$ and the maximal $\max H_{arc}$ value of the horizontal force are determined so that the arch is stable.
2. The value $\min H_{arc}$ is considered as the effective horizontal force of the central arch. The minimal and maximal horizontal forces $\min H_p$ and $\max H_p$ of the side arch are calculated in order that the entire system is stable. For this purpose, an iterative procedure is performed, in which the value of the horizontal force H_p is varied and the equilibrium of the piles and the buttresses is simultaneously examined. The procedure is terminated when the extreme values of H_p that satisfy the equilibrium of all the structural elements are found.
3. The same procedure is repeated for the maximal value $\max H_{arc}$ and for a range of values of H_{arc} between $\min H_{arc}$ and $\max H_{arc}$.

As a result of this procedure, all the possible combinations for the forces H_{arc} and H_p that assure the stability of the system are determined. It is important to note that the extreme values of $\min H_p$ and $\max H_p$ calculated with this procedure are not the same as the ones obtained by the consideration of each arch separately.

The results for the two extreme values of H_{arc} are presented in the Fig. 3 where the following convention is introduced: the dotted and the continuous line respectively correspond to the lines of thrust created using the minimal and maximal values of the horizontal forces applied to the arches respectively.

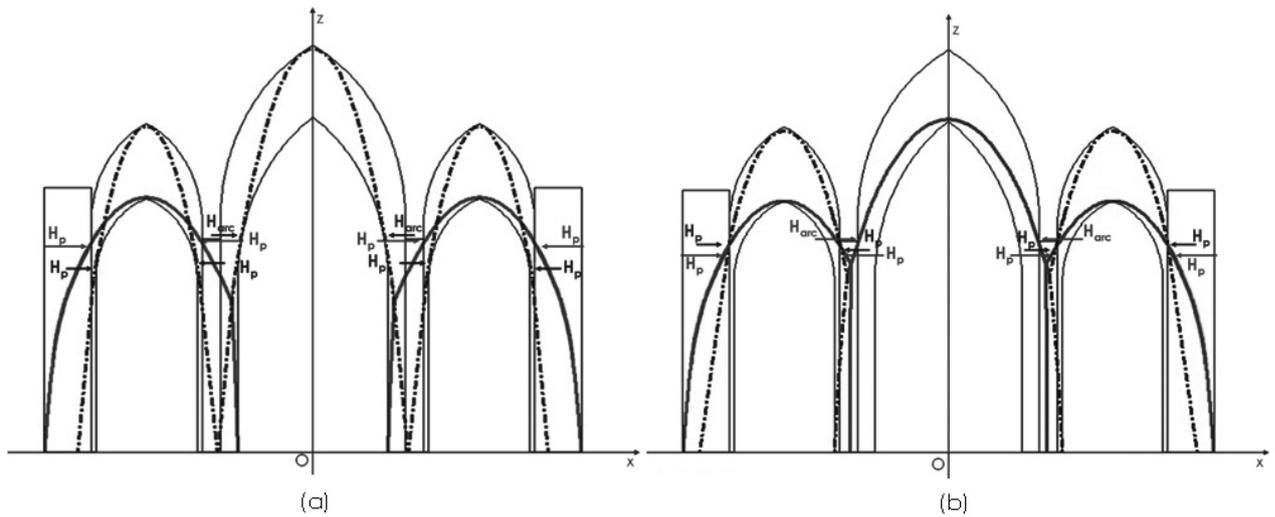


Figure 3: Thrust lines of the system for the case of loading under self-weight for a) minimal horizontal force ($\min H_{arc}$) to the central arch, b) maximal horizontal force ($\max H_{arc}$) to the central arch

Consideration of the stability of joints

As an example, the joint between the central and the right side arch will be presented (Fig. 4). We consider the reaction R_{arc} of the central arch to the pile and the reaction R_p of the right arch to the pile. We denote as M and N their application points respectively. The point I_1 lies on the intersection of the tangents of the two thrust lines. The block ABF_1D_1 is divided in two blocks and we consider that the block $AC_1I_1D_1$ is the influence area of the central arch and the block $C_1BF_1I_1$ the influence area of the right arch. As influence area of an element we define the area of the joint in which the construction of the thrust line of the element is extended into.

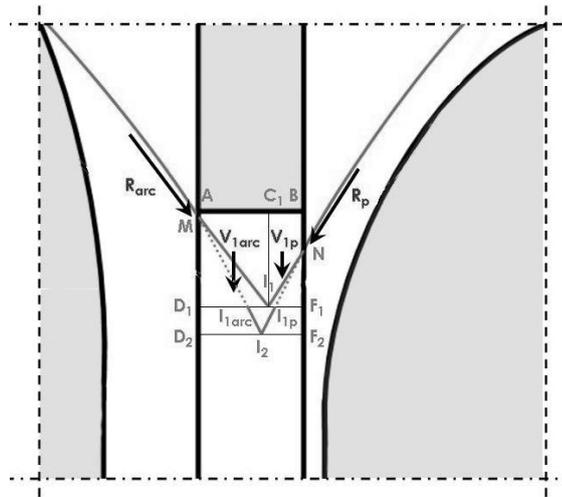


Figure 4: Resolution of the joint stability for the lines of thrust of the central arch and the right side arch

In order to determine the way in which the thrust line of each element is extended in the block ABF_1D_1 we consider the reaction force of each arch and the weight of each influence area. The problem requires thus the determination of the point of intersection I of the three thrust lines (central arch, right arch and total weight of the bloc ABF_1D_1). It is the point where equilibrium is satisfied between the reaction forces and the weight of the joint bloc. We note that the weights of the influence areas are functions of the position of the point I , the determination of which depends again on the weight of the influence area. So the determination of the point I is achieved through an iterative procedure: a prediction of the position of I is initially introduced, the equilibrium is checked and if it is not satisfied, a correction of the position of the point I is made.

The required iterative procedure is explained in the following. After finding the intersection point I_1 (intersection point of extended thrust lines of the two arches), for the influence areas, $AC_1I_1D_1$ and $C_1BF_1I_1$, we calculate the direction and application point of the resultant reaction force necessary for moment and force equilibrium in each influence area (considering loading under unit weight and under the reaction force coming from the corresponding element). Naming I_{1arc} and I_{1p} the points of equilibrium of $AC_1I_1D_1$ and $C_1BF_1I_1$ respectively, and extending the lines MI_{1arc} and NI_{1p} we find the point I_2 that constitutes a new prediction for the intersection point I . The procedure is repeated until the distance between I_1 and I_2 lies within a prescribed tolerance.

Seismic loading

The approach presented above can also be extended to horizontal loading and in particular seismic loading. In this last case, inertia forces (they arise due to the vibration of the structure) may be considered as acting pseudo-statically at the discretized nodes of the structure. Generally speaking, dynamic loading can increase local stresses causing crushing failure of the masonry, and can cause vibrations which may increase the possibility of sliding. Even so, the no-tension material assumptions (Heyman 1966) introduced in the previous case will also be considered applicable for seismic loading conditions.

To this end, a horizontal seismic force will be considered at the centre of gravity of each block i obtained by the discretization of the structure. This force is equal to:

$$H_{s,i} = K_s m_i g \tag{4}$$

where $H_{s,i}$ = the seismic force in the block i , K_s = the seismic coefficient, m_i = the mass of each block i and g = the ground acceleration.

The horizontal seismic force can be composed with the weight of each block to give a resultant force with a direction parallel to an inclined line and not a vertical one. The angle of inclination φ of this line is equal to:

$$\varphi_i = \tan^{-1} \frac{H_{s,i}}{V_i} = \tan^{-1} \frac{K_s m_i g}{m_i g} = \tan^{-1} K_s \tag{5}$$

where V_i = the weight of each block i .

For the construction of the thrust line, the same procedure as in the case of static loading can thus be followed with the difference that the possible locus ε' of each nodal point is rotated by an angle φ as shown in Fig. 5. Similar approximations for pseudo-static seismic loading using graphic statics have been proposed by Block et al (2006b).

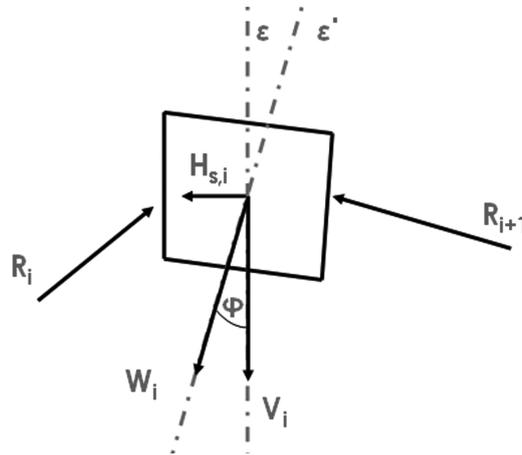


Figure 5: Change of coordinates in the case of seismic loading

The results for the two extreme values of H_{arc} are presented in Fig. 6 for a seismic loading with $K_s=0.1$. The values H_p and H_{pl} of the horizontal forces acting in the right and in the left extremity of each side arch in this non symmetric case of loading are in each case the least favorable for the stability of the system. The method used for the blocks i of the structure was also used for the consideration of the stability of joints by applying the seismic force $H_{s,i}$ in each influence area.

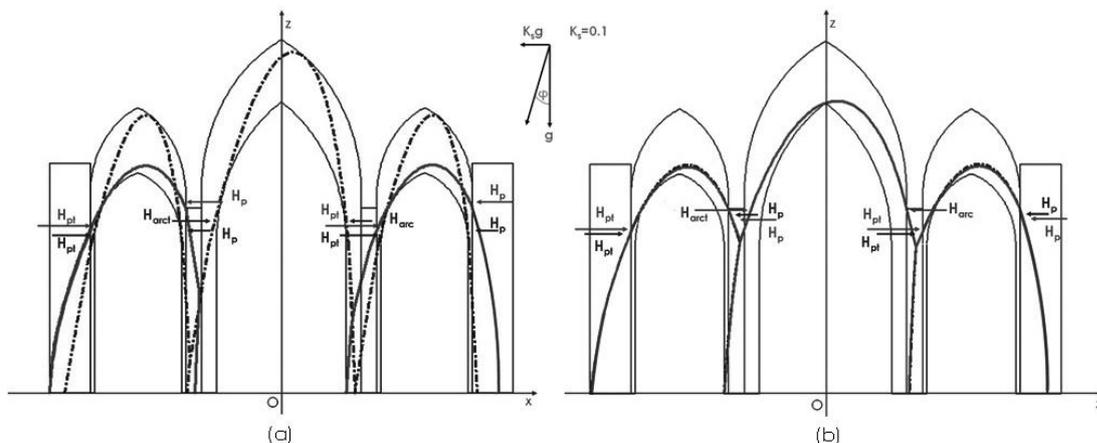


Figure 6: Thrust lines of the system for the case of seismic loading ($K_s=0.1$) for a) minimal horizontal force (min-Harc) to the central arch, b) maximal horizontal force (maxHarc) to the central arch

The results presented above allow defining the domain of all the admissible combinations of the horizontal forces H_{arc} , H_{pD} and H_{pR} for the system. Approximate representations of this domain for dead loading were introduced by Smars (2000) and Block et al. (2006a). Comparisons with these works are not possible as the system presented in this paper has a different geometry. The domain of the system is defined as the convex envelope of the combinations of H_{arc} , H_{pD} and H_{pR} that assure the stability of all the elements of the system; this is an immediate consequence of the static theorem of limit analysis. The domain of equilibrium of the system presented in this paper is shown in Fig. 7 where five cases are shown. The dotted line corresponds to the domain of the system in the case of loading under self-weight and the continuous lines correspond to domains obtained for seismic loading of increasing maximum acceleration. The figure shows the reduction of the domain of the admissible combinations of the horizontal forces as the seismic accelerations rises. The domain for $K_s=0.17$ is reduced to a single point; this situation can be thought of as an ultimate limit state of the system for which stability is attained only under one single combination of horizontal forces.

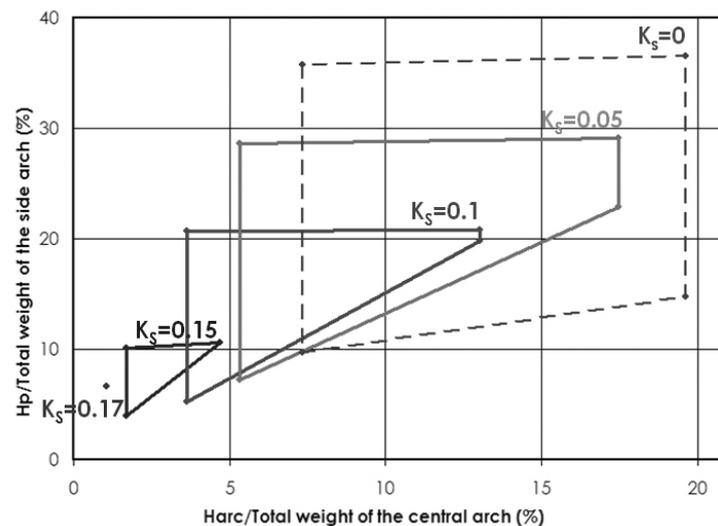


Figure 7: Domains of equilibrium of the system for loading under self-weight (dotted line) and for seismic loading (continuous lines)

The following remarks can be made with respect to the presented results. When applying seismic loading to the structure (Fig. 6), the change in the position of the critical points for hinge formation can be observed. The susceptible points in the intrados and the extrados of the arches are different from those obtained for loading under self-weight (Fig. 3). Moreover, in the case of maximal thrust in the central arch, the range of admissible horizontal forces in the side arches is much smaller than in the case of minimal thrust. As the seismic acceleration increases this phenomenon becomes more pronounced (Fig.7) up to the point that the two extreme values for the side arches coincide for K_s approximately larger than 0.15. In other words, the domain of equilibrium becomes triangular before shrinking down to a single point.

CONCLUSIONS

In this article we presented two modeling tools for the assessment of the mechanic behavior of masonry structures under in-plane loading based on limit analysis. Simple hypotheses have been introduced with the objective of constructing thrust lines for two-dimensional structural systems directly obtained from three-dimensional restitutions automatically created through few geometric parameters. Through the easy-to-use yet comprehensive thrust line method, the uncertainties related to determining masonry mechanical parameters are overcome and the stability domain of the structure together the regions of possible damage can be visualized.

The newly presented methodology for treating the stability of joints has allowed for a coherent consideration of the equilibrium of the entire structure and has highlighted the coupling effect between the different structural elements. Analyses of pseudo-static seismic loading have led to an estimation of the maximum earthquake severity (expressed through seismic acceleration) that the structure can sustain.

From a theoretical point of view, the tools were based on the no-tension material concept and the related assumptions introduced by Heyman (1966). Even though these assumptions may become questionable, especially for seismic loading conditions, the utility of the method resides in the fact that it can easily be applied to a large number of structures in order to provide a preliminary stability assessment and to indicate for which structures a more sophisticated approach should be adopted.

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